Physics of rare events

Introduction and quick overview

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1 Dark Matter Direct Detection

Dark Matter (DM) is ubiquitous. It fills galaxies and clusters of galaxies, and governs their dynamics. It has been shaping the evolution of the Universe since at least the time of CMB formation (recombination, 380 kyr after the Big Bang) and it provides the scaffolding of the Large Scale Structure of which the Universe is made. It constitutes 25.8% of the total energy content of the Universe and about 84% of the matter content only. Yet its nature is unknown.

We do know that **Dark Matter must**: i) behave like a *corpuscle* (its density must dilute as $1/V$ where V is the volume of the Universe, as the Universe expands), ii) be *very feebly interacting* (e.g. Standard Model weak interactions) with ordinary matter and also be not too strongly interacting with itself, iii) be *cold* (non-relativistic since at least the start of structure formation), iv) be *stable* (or at least with a lifetime much longer than the age of the Universe). Within these very broad contours, many things can fit. For instance, the **mass of the DM** is practically not determined at all. The hard work of hundreds of theorists in the past decades has managed to restrict its value to a range of about 92 orders of magnitude (see fig. 1). Even neglecting the extremes, the range in which DM is of interest to particle physics experiments extends from the KeV to hundreds of TeV. This breadth must of course be kept in mind when thinking about detecting DM.

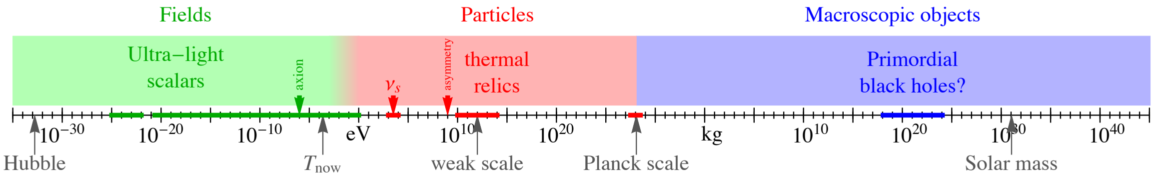


Figure 1: *Possible range for the DM mass and some notable candidates.*

Direct detection experiments search for signals due to scatterings of galactic Dark Matter particles on atomic nuclei or on electrons.

The most studied case is the one in which the struck body is a **nucleus**. In that case, the energy transferred from DM to the nucleus is comparable to the kinetic energy of the DM–target two body system, $K = \mu v^2/2$, where $\mu = M M_T/(M + M_T)$ is the reduced mass of the DM–target system, and $v \approx 10^{-3}\ c$ is the relative velocity, dictated by the typical speed of DM in the galactic halo of the Milky Way.

Numerically, $K \approx 25$ keV if DM has a mass comparable to heavy nuclei, $M \approx M_A \approx 100\,\mathrm{GeV}$. This is small enough that the scattering is elastic, with the nucleus remaining in the same state before and after the collision. At the same time K is also large enough that it can be detected. The expected number of events per unit of time, assuming that DM particles have velocity v, is given by

$$\text{event rate} = N_T \frac{\rho_\odot}{M} v \sigma_A \approx \frac{1}{\mathrm{yr}} \times \frac{M_T/A}{\mathrm{kg}} \frac{\sigma_A}{10^{-39}\,\mathrm{cm}^2} \times \frac{\rho_\odot}{0.3\,\mathrm{GeV/cm}^3} \times \frac{v}{200\,\mathrm{km/s}} \times \frac{100\,\mathrm{GeV}}{M},$$

where $M_T = N_A M_A$ is the mass of the detector composed of N_A nuclei with atomic number A and mass $M_A \approx A m_N$, where $m_N = 0.939$ GeV is the nucleon mass, and σ_A is the DM cross section for scattering on the *nucleus*. The latter is usually converted to a scattering cross section for DM scattering on a *nucleon*, σ_N. Quite often such scattering does not depend on nuclear spin. Then, the main observable to be measured is the spin-independent cross section for DM scattering on nucleons, $\sigma_N = \sigma_{\rm SI}$. With 1 event per year or less in typical conditions, the process does merit its label of **'physics of rare events'**. Fig. 2 shows the (theorist's simplified view of the) recent progress in Direct Detection limits.

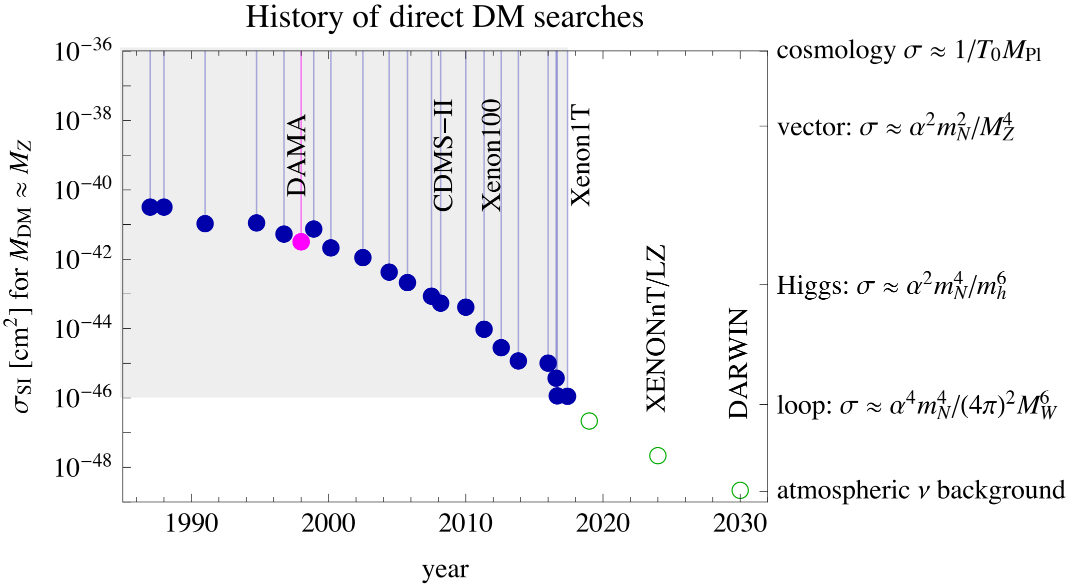
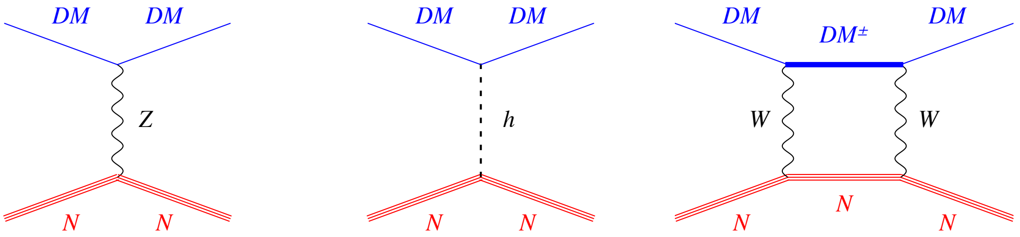


Figure 2: *A theorist's view of the progress in DM Direct Detection in the past decades, compared to the suggestions from theory (right edge of the plot). The plot applies to 'heavy' DM, with $M_{\rm DM} \approx M_Z$, and to spin-independent scattering.*

What does theory predict? Figure 2 also shows the expected DM scattering cross sections for several examples of DM interactions. The corresponding diagrams are depicted in fig. 3.

- ◇ Tree-level Z exchange would give $\sigma_{\rm SI} \sim \alpha_Y^2 Y_{\rm DM}^2 m_N^2/M_Z^4 \sim 10^{-38}\mathrm{cm}^2$ and is excluded, unless DM has a small (effective) hypercharge $|Y_{\rm DM}| \lesssim 10^{-4}$.
- ◇ Tree-level Higgs exchange would give $\sigma_{\rm SI} \sim \alpha_{\rm DM} m_N^4/M_h^6 \sim 10^{-43}\mathrm{cm}^2$ where $\alpha_{\rm DM} \sim y_{\rm DM}^2/4\pi$ if DM is a fermion with a Yukawa coupling $y_{\rm DM}$, constrained to be smaller than about 0.1.

- 1-loop electroweak diagrams give $\sigma_{\rm SI} \sim \alpha_2^4 m_N^4/(4\pi M_W^3)^2 \sim 10^{-48}{\rm cm}^2$, which is still allowed by experimental constraints.



tree, vector

$$\sigma_{\rm SI} \approx \frac{\alpha^2\, m_N^2}{M_Z^4}$$

tree, scalar

$$\sigma_{\rm SI} \approx \frac{\alpha^2\, m_N^4}{M_h^6}$$

loop

$$\sigma_{\rm SI} \approx \frac{\alpha^4\, m_N^4}{M_W^6}$$

Figure 3: *Typical diagrams and scattering cross sections.*

So summarizing, theory predicts first of all a huge variability of cross sections, spanning orders of magnitude (not very differently, conceptually, from the 'prediction' for the DM mass). Theory does *not* predict that we are now cornered to an improbable final portion of the parameter space and that we are forced to recur to loops to hide in the last available slice, because a 1-loop interaction is not more preposterous than the tree-level Z-exchange. For instance, it suffices that the DM candidate be a Majorana particle for the Z-exchange to exactly vanish. It suffices that the DM particle have hypercharge $Y = 0$ for the higgs-coupling to exactly vanish. In that case the cross-section of the 1-loop diagram is the largest contribution and it should be the target of the searches. On the other hand, it is fair to say that experiments have covered a lot of the ground suggested by theory, without finding anything, and this should be seriously pondered. Actually, most of the above discussion implicitly addresses WIMPs, so it is interesting to discuss them explicitly.

WIMPs. A caveat: the definition of WIMPs (Weakly Interacting Massive Particles) in the community is slightly ambiguous. Does the W stand for Weak in the sense of $SU(2)_{\rm L}$ Standard Model interactions or does it stand for generically feeble interactions, similar to those of the weak SM force? The (very well known) considerations that follow apply strictly speaking to the former definition, but they can be extended to the more general case.

The **motivations** for WIMPs rest on two main pillars:

1. Relic abundance: a particle with these properties is produced in the early Universe in the right amount, thanks to the mechanism of thermal freeze-out. This is such a highly non-trivial coincidence that it is sometimes called a miracle. Other production mechanisms typically do not provide indications of this sort.

2. Naturalness: the need and the expectation of New Physics at the TeV scale in order to stabilize the mass of the higgs boson in the Standard Model carries with it as a byproduct the existence of DM candidates.

While the second pillar is nowadays seriously questioned by empty-handed searches at the LHC and elsewhere, the first motivation still stand. Hence the often-heard shortcut that WIMP DM is dead because the LHC has found no New Physics (yet) amounts to telling half the story, to say the least.

About WIMPs, it is interesting to point out that 'recent' (last 10 years) developments have pushed the **plausible mass scale to the multi-TeV** region. Not because the smaller masses have been probed already and nothing has been found (which is also in part true, as we have mentioned above), but because of theory motivations. The case of **pure WIMPs**, defined as particles that have only $SU(2)_{\rm L}$ interactions, for instance 'pure Wino DM' or 'pure 5-plet DM', is apparent: they feature thermal masses of the order of 3 TeV and 11 TeV respectively. The reason is essentially that their annihilation cross section in the Early Universe is large (because of co-annihilations and because of phenomena such as the Sommerfeld enhancement and the formation of unstable bound states), therefore their relic number density is small hence they have to be heavy to reproduce the needed relic abundance. In other words, WIMPs motivated by relic abundance considerations (pillar 1) naturally have multi-TeV masses.

The **light DM option** (few GeV and sub-GeV) is comparatively much more recent than WIMPs and, it is fair to say, less well theoretically motivated. It is often said that previous claims of discovery/anomaly (Dama/Libra, Cdms) point to a DM with a mass of a few GeV, but the corresponding scattering cross sections (if on nuclei) are severely challenged by subsequent experiments and the theory models that can reconcile the two, if they exist, are very baroque. It is also often said that dark sector models *allow* for sub-GeV DM, which is true, but they do not *necessarily* point to that range. The main argument for light DM is therefore of the 'why not?' sort. Which is however fully legitimate after all.

In this regime of mass, DM cannot produce any significant nuclear recoil so the energy deposition (i.e. detection) process is via **scattering on electrons**. The rule of thumb is that a DM particle of mass M can transfer to the electron a maximum energy E_e equal to the DM kinetic energy

$$E_e \leq \frac{1}{2} M v^2 \approx 0.2 \text{ eV} \times \frac{v}{200 \text{ km/s}} \times \frac{M}{\text{MeV}},$$

so that it can free an electron with binding energy ΔE_B if

$$M \gtrsim 5 \text{ MeV} \times \frac{200 \text{ km/s}}{v} \times \frac{\Delta E_B}{1 \text{ eV}}.$$

This conveys the idea that detectors that are sensitive to single electrons and which use elements with typical binding energies in the few eV range can probe down to the ∼MeV DM mass limit.

2 Neutrinoless Double Beta Decay

Neutrino physics shares many points of contact with the physics of Dark Matter, both from the phenomenological and the theoretical point of view. Neutrinos are also ubiquitous in the Universe, they are also a very feebly interacting particle (actually weakly interacting in the technical sense mentioned above) and they offer a **window to New Physics Beyond the Standard Model**.

A very significant amount of progress has been made in neutrino physics in the past decades, with ground breaking discoveries and precise measurements (contrast that to Dark Matter physics, by the way), but still many **questions remain unanswered**. They in particular include:

(○) Are neutrinos Majorana or Dirac particles? I.e. do particle and antiparticle coincide in the case of neutrinos (contrary to all the other known fermions of the Standard Model) or not?

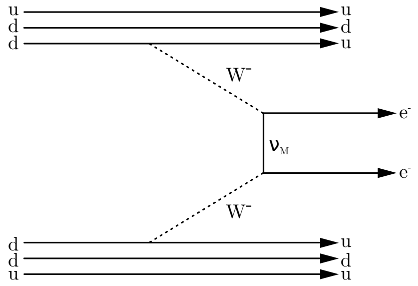
(○) What is the absolute mass scale? From oscillation phenomena we can measure the differences in mass, but the offset value, on top of which these differences sit, is not known yet.

(○) What is the mass hierarchy? From oscillations we can measure the mass splittings between the neutrino mass eigenstates, but we do not know their relative ordering.

If neutrinos are Majorana particles, then **neutrinoless double beta decay** (‘$0\nu\beta\beta$’)

$$(A, Z) \rightarrow (A, Z+2) + 2e^-$$

can happen, according to the diagram in fig. 2 (the inverse is not true: $0\nu\beta\beta$ could occur also via other, arguably less interesting mechanisms).



The most striking feature of such a process is of course that it **violates (electron) lepton number** by two units. The discovery of $0\nu\beta\beta$ would therefore demonstrate that lepton number is not a valid symmetry in Nature. Via the connection to baryon number, this could open the way to the possibility that neutrinos played a role in the establishment of the matter-antimatter asymmetry in the Universe (baryogenesis via leptogenesis). No fundamental process has ever been observed where lepton number is violated, and we have no deep reason why it should be preserved. In the Standard Model, however, this is the case. Lepton number L (and baryon number B) are accidental symmetries, preserved due to the specific particle content of the theory. As a consequence, the $0\nu\beta\beta$ transition is completely forbidden.

The crucial parameter involved in the $0\nu\beta\beta$ process (if mediated by Majorana neutrinos) is the quantity called '**effective Majorana mass**'

$$m_{\beta\beta} = \left| \sum_{i=1,2,3} U_{ei}^2 \, m_i \right|$$

where m_i are the masses of the neutrino mass eigenstates ν_i and U is the unitary matrix that diagonalizes the neutrino mass matrix in the Lagrangian and connects mass eigenstates and flavor eigenstates ν_ℓ

$$\nu_\ell = \sum_{i=1,2,3} U_{\ell i} \, \nu_i .$$

Defining the mass parameter $m_{\beta\beta}$ allows to **connect to other** mass-related **observables of the neutrino sector**. Fig. 4 represents such a connection.

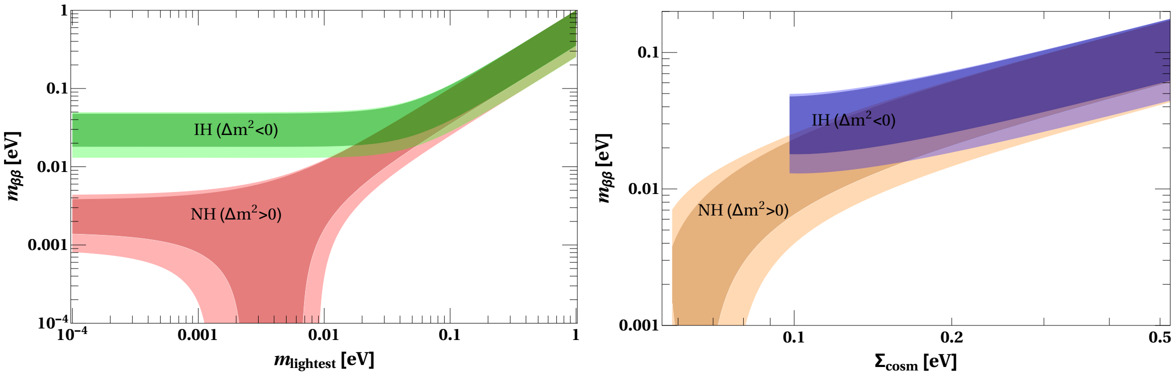


Figure 4: *Predictions of $m_{\beta\beta}$ from neutrino oscillations as a function of the lightest neutrino mass (left) and of the sum of neutrino masses, relevant in cosmology. Figure from Dell'Oro et al., arXiv:1404.2616, updated and adapted from Strumia and Vissani, arXiv:hep-ph/0606054.*

Although the absolute neutrino mass scale is still unknown, oscillation experiments allow to measure the squared mass splittings between the three active neutrinos

$$\begin{aligned} \delta m^2 &= m_2^2 - m_1^2 = (7.54 \pm 0.26) \times 10^{-5}\ \text{eV}^2, \\ \Delta m^2 &= m_3^2 - 1/2(m_1^2 + m_2^2) = (2.44 \pm 0.08) \times 10^{-3}\ \text{eV}^2, \end{aligned} \tag{1}$$

the former mostly measured by solar oscillation and the latter by atmospheric ones. The sign of δm^2 can be determined by observing the effect of matter oscillations in the Sun, and it turns out that $\delta m^2 > 0$. On the other hand, the sign of Δm^2 is unknown and the challenge is out to measure it. If $\Delta m^2 > 0$ we talk of 'normal hierarchy' (NH) while $\Delta m^2 < 0$ is 'inverted hierarchy' (IH). From cosmology one can instead measure the sum of all neutrino masses

$$\Sigma_{\text{cosm}} = m_1 + m_2 + m_3 . \tag{2}$$

Thanks to these interplays, the quest for $0\nu\beta\beta$ is connected to all the unanswered questions in neutrino physics, and ultimately to the role of neutrinos in the Universe (both in the sense of their cosmological mass and in the sense of their possible role in generating the matter-antimatter asymmetry).